EXPERIMENTS ON THE CELLULAR STRUCTURE IN BÉNARD CONVECTION

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Abstract-Natural convection in a cylindrical cell was investigated experimentally using two different fluids heated from below. On the whole, good agreement was obtained with many theoretical predictions. In silicone oil, the physical properties of which are relatively insensitive to temperature, the fiow pattern was found to consist of concentric rolls in the case of buoyancy-driven convection, and of hexagonal cells for surface-tension-driven flow. These observations are consistent with those reported recently by Koschmieder. With Aroclor, the viscosity of which is highly temperature dependent, rolls appeared for liquid depths ≥ 6 mm and hexagons for depths ≤ 5 mm. Rather than transform into rolls as some theories would have predicted, these hexagons, which were somewhat smaller than expected, were found to remain stable over a range of temperature differences extending up to at least three times the critical. This suggests that presently available non-linear analyses should not be extrapolated much beyond the critical point, where in fact they are meant to apply.

1. INTRODUCTION

A LARGE number of papers have been published on the subject of convection in horizontal liquid layers heated from below. This phenomenon, due generally to buoyancy forces within the layer and/or to surface-tension-gradients along the upper surface, arises whenever the temperature gradient across the liquid exceeds a certain critical value. The latter can be accurately predicted using the classical techniques of linear hydrodynamic stability analysis $\lceil 1-3 \rceil$ but, as is well known, the possible flows at the point of instability form an infinitely degenerate set. Consequently, the very striking and regular flow patterns that emerge following the onset of convection can only be accounted for on the basis of a non-linear theory.

Some of the recent theoretical results on the question of the preferred flow structure near the critical point are as follows:

(a) Motion induced by surface-tension gradiients leads to a pattern of hexagons [4].

(b) In buoyancy-driven convection, twodimensional rolls remain the only stable mode for fluids having temperature *independent* physical properties [S].

(c) Again in buoyancy-driven convection but for fluids having temperature dependent physical properties, the motion is of the hexagonal type for a range of Rayleigh numbers near the critical [G-9]. Upon further increase in the Rayleigh number, these hexagons transform into rolls [9].

Although there exists ample experimental evidence in support of the first two theoretical predictions $\lceil 10-13 \rceil$, no experimental test of the third prediction appears to have been attempted to-date. The present note is intended to supply this information.

Following is a brief description of the experimental apparatus and of the results which were obtained.

2. APPARATUS AND EXPERIMENTAL PROCRDURE

The apparatus was similar to Koschmieder's [11]. A $\frac{1}{2}$ in. thick copper plate, $8\frac{1}{16}$ in. in

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diameter, served as the bottom and was heated from below with water impinging on its center and spreading to the sides between the copper plate and a lucite disk. The lateral wall consisted of a $\frac{1}{4}$ in. thick lucite ring which was attached to the copper plate. A polished glass plate, $\frac{1}{8}$ in. thick and $7\frac{1}{2}$ in. in diameter, served as the lid and was cooled from above with water impinging on its center and spreading to the sides under another lucite disk. Using a piston type assembly, the glass plate could be held to any desired position parallel to the copper plate.

The glass and copper plate temperatures were set by circulating the water through constant temperature baths, controlled to $+0.05^{\circ}$ C. The temperature difference was determined with thermometers $(+0.05^{\circ}C)$ which measured the temperature of the water pumped directly to the plates. The cold water temperature was kept constant at 22.5° C, and the flow rates were approximately 400 cm^3/min .

Fine aluminum powder was suspended in the fluid to observe the convective motions. Since the flakes align themselves with the flow and reflect light, bright areas refer to horizontal motions whereas dark areas correspond to vertical motions. The flow pattern was recorded by taking pictures either through the piston assembly whenever the experiment was such that the liquid surface touched the glass plate, or with the glass plate removed for less than 10 s if the experiment was performed with an air-gap separating the glass plate and the liquid. The copper plate was blackened to improve the photographic contrast. Finally, the positions of the glass and copper plates as well as that of the liquid surface were measured to $+0.001$ cm by means of a cathetometer; hence, the depth of the fluid layer could be determined to within $+0.002$ cm.

In order to effect a comparison between the theoretical predictions and the experimental results it was necessary to determine the geometry and dimensions of the "cells", as well as the temperature gradient within the liquid layer at the onset of convection. As can be seen

from the photographs to be discussed shortly, the former could be obtained without difficulty. However, since a well defined flow pattern did not set in until the temperature difference across the layer exceeded the critical, usually by about $10-20$ per cent but occasionally by $40-50$ per cent, values for the cell sizes to be reported are not exactly those pertaining to the critical point.

The determination of the critical temperature gradient was also subject to uncertainty. To begin with, since it was not possible in the present set-up to measure directly the temperatures of the two plates, the assumption had to be made that these were approximately equal to those of the appropriate constant temperature baths. In addition, as was also done by Koschmieder [12], account had to be taken of the temperature drop across the glass plate and, whenever applicable, across the air-gap. For example, with ΔT denoting the temperature difference between the baths, the temperature drop, ΔT^* , across the liquid layer at the onset of convection was calculated using the expression

$$
\frac{\Delta T}{\Delta T^*} = 1 + \frac{k l_1}{k_1 l} + \frac{k l_2}{k_2 l} \tag{1}
$$

where *k* and *l* are, respectively, the thermal conductivity and thickness of the appropriate substance (No subscript refers to the fluid, subscript 1 to the glass and subscript 2 to the air). In the presence of an air-gap ΔT^* was often 50 per cent or more lower than the measured temperature difference ΔT .

In computing the theoretical values for the critical temperature difference ΔT^* and the critical wave-number α , care had to be taken to apply the proper boundary conditions. Due to the very large thermal conductivity of the copper plate, it seemed safe to assume that the lower boundary was isothermal and rigid. On the other hand, at the upper surface of the liquid, the thermal boundary condition was of the radiation type

$$
\frac{\partial \theta}{\partial z} + L\theta = 0 \tag{2}
$$

where θ is the temperature fluctuation, suitably normalized $[3]$, z is the dimensionless coordinate in the vertical direction, and

$$
L^{-1} = \frac{kl_1}{k_1 l} + \frac{kl_2}{k_2 l} \tag{3}
$$

Equations (1) and (3) apply only as long as the air layer remains stagnant up to the critical point However, since the thickness of the airgap was always very small, O(1 mm), this was shown theoretically to have been always the case.

Actually, the chief difficulty in comparing the theoretical with the experimental results arose from the fact that the point at which convection set in could not be determined unambiguously. To begin with, any visual determination of the point of instability is somewhat inaccurate because it requires that the fluid motions be of sufficient magnitude to be observable. In our case, though, the most serious problem occurred near the lucite ring forming the lateral wall. Apparently, owing to ineffective thermal insulation, a temperature difference set in between this ring and the adjacent fluid which, in turn, gave rise to a localized convection motion. Thus, a single roll cell frequently appeared at this outer rim even though the rest of the fluid remained motionless. However, since this phenomenon was clearly due to a wall effect which is not accounted for by any theoretical analysis, the appearance of this localized motion was not taken to indicate the presence of the theoretically expected convective flow. Instead, the experimentally determined point of instability was chosen to be that at which motion was observed within the liquid layer away from the rim. It is evident, though, that the experimentally obtained values for the critical temperature difference ΔT^* are subject to some uncertainty which, generally, was found to be O(10 per cent).

3. **EXPERIMENTAL RESULTS**

The first series of experiments was performed with Dow Corning 200 fluid, a silicone oil, having kinematic viscosity $v = 0.50$ cm²s⁻¹ at

24°C, density $\rho = 0.963$ gcm⁻³, thermal expansion coefficient $\beta = 0.96 \times 10^{-3}$ °C⁻¹, specific heat $c_p = 0.35$ cal $g^{-10}C^{-1}$, thermal conductivity $k = 0.37 \times 10^{-3}$ cal cm⁻¹s⁻¹°C and slope of the surface tension vs. temperature curve $-\sigma = 0.58$ dyn cm^{-1°}C⁻¹. Data were taken both with and without an air gap. The results of two representative runs are shown in Table 1. Here ΔT_c and ΔT refer to the temperature difference at, respectively, the critical point and at the point at which *a* was measured.

In all cases, it was found that the flow pattern just beyond the critical point was rather irregular. For example, in a few experiments it consisted of a mixture of concentric rolls near the rim and hexagonal cells near the center. This is in qualitative agreement with the point of view expressed by Koschmieder [ll], by Chen and Whitehead [13] and by Davis [14] according to which the flow structure near the critical point is very sensitive to secondary effects, such as the shape of the container, the rate of heating small non-uniformities in the boundary conditions etc. However, when the temperature difference was increased to a value 30-50 per cent larger than the critical, a regular and highly reproducible pattern emerged which, as shown in Figs, 1 and 2, consisted of concentric rolls in the case of a rigid upper surface, and of hexagons whenever the liquid layer was in contact with an air-gap. These patterns persisted up to values of the temperature difference as large as three times the critical, beyond which the experiments were discontinued.

Referring to Table 1, we see that there is very good agreement between the experimental and the theoretical results, the latter having been computed from the solutions given by Sparrow et al. [15] and by Nield [3] for, respectively, a rigid and a free upper surface. In experiment no. 1, seventeen rolls were observed at a Rayleigh number $R = 2900$ thus giving a value for the wave number $\alpha = 2.78$ (eighteen rolls would have increased α to 2.94, in almost perfect agreement with the theoretical value at the critical point). As for the results from experiment no. 2,

by Scalon and Segel [4] that convection driven that this discrepancy may be due to the presence by surface-tension gradients leads to hexagonal of secondary effects, such as the shape of the flow patterns. To be sure, in the present case container, which, as mentioned earlier, exert a convection was due to the combined effect of strong influence on the flow pattern near the buoyancy forces and surface-tension gradients ; critical point. Thus, the use of a cylindrical dish however, the experimentally determined Ray- may have been responsible for the total absence
leigh number at the critical point was only about of hexagons beyond the critical point. To be l/3 that required theoretically to account for sure, as was the case earlier with the silicone oil, convection in the absence of surface-tension a few irregular cells did appear at the onset of effects, whereas the Marangoni number was convection, however, following an increase in generally about 2/3 that given by Pearson [2] the temperature difference, the flow quickly and by Nield [3] as the criterion for surface- developed into a pattern of regular concentric tension-driven flow in the absence of buoyancy rolls such as that shown in Fig. 3. In all other forces. It would appear therefore that, under the respects, the results from experiment no. 3 were conditions of the present experiments with an in accord with the theoretical predictions and air-gap, the Row was due primarily to the action were similar to those of experiment no. 1. Rolls of surface-tension gradients. were also formed with a liquid depth $l = 6.0$ mm.

All the results discussed above are in essential agreement with those reported previously by Koschmieder [11, 121.

The novel feature of the present work involved experiments with Aroclor 1248, an aromatic hydrocarbon, the viscosity of which is highly temperature dependent Its properties are as follows: $k = 0.23 \times 10^{-3}$ cal cm⁻¹s⁻¹ ${}^{\circ}C^{-1}$, $\rho = 1.45$ g cm⁻³, $\beta = 0.70 \times 10^{-3} {}^{\circ}C^{-1}$, $c_n = 0.275 \text{ cal}^{-1\circ} \text{C}^{-1}$, and a kinematic viscosity v which, in the temperature range $20-50^{\circ}$ C, can be represented by

$$
v = 2.21 \times 10^{-3} \exp \left\{ \frac{34}{t + 4.836} \right\} \text{cm}^2 \text{ s}^{-1},
$$

$$
t = \frac{T - 22}{10} \tag{4}
$$

with T being the temperature in $^{\circ}$ C. Thus $v = 3.0, 0.87$ and 0.38 cm²s⁻¹ at, respectively, 20°C 30°C and 40°C.

Again experiments were performed with and without an air-gap.

As shown in Table 1, convection in a 6.97 mm deep layer in contact with the glass top led to rolls. This is in disagreement with the criterion developed by Palm et al. [9] according to which hexagons should have been observed within the

these seem to verify the theoretical prediction Rayleigh number range 1650-2150. It is believed of hexagons beyond the critical point. To be

> When the depth was decreased to 3.64 mm, the flow pattern was found to consist of regular hexagons, shown in Fig. 4, which formed as soon as convection set in and which persisted until the end of the experiment at a temperature difference approximately twice the critical. A similar result was obtained in the case of a 4G mm depth where the hexagons remained stable up until the very end of the experiment, 13 h after the onset of the convection, when the temperature difference had reached a value approximately three times the critical. Stable hexagons were also formed with $l = 5.0$ mm. Since it is known that the use of large rates of heating may also lead to hexagons even in a fluid having constant physical properties [16], care was taken to employ a rate of heating no greater than 1.5° C h⁻¹ which, according to the criterion developed by Krishnamurti [16], is much too small to account for the appearance of any hexagons in the present experiments.

> The existence of these hexagons is of course in agreement with the theoretical predictions, however, their failure to transform into rolls at $R \approx 3000$ (for $l = 3.64$ mm) or at $R \approx 2500$ (for $l = 40$ mm) runs counter to the theory given by Palm *et al.* [9]. This is not too surprising, though, since Palm's analysis, which applies only close

- FIG. 1. Convection in silicone oil. Upper surface in contact with glass. $l = 4.95$ mm, $\Delta T/\Delta T_c = 1.53$.
- FIG. 2. Convection in silicone oil. Upper surface in contact with an air gap. $l = 5.05$ mm, $l_2 = 0.99$ mm, $\Delta T/\Delta T_c = 1.48$.
- FIG. 3. Convection in Aroclor. Upper surface in contact with glass. $l = 6.97$ mm, $\Delta T/\Delta T_c = 1.51$.
- FIG. 4. Convection in Aroclor. Upper surface in contact with glass. $I = 3.64$ mm, $\Delta T/\Delta T_c = 1.26$.
- FIG. 5. Convection in Aroclor. Upper surface in contact with air. $l = 6.92$ mm, $l_2 = 1.01$ mm, $\Delta T/\Delta T_c = 1.09$.
- FIG. 6. Convection in Aroclor. Upper surface in contact with air. $I = 3.99$ mm, $I_2 = 1.01$ mm, $\Delta T/\Delta T_c = 1.15$.

to the critical point, should not be invoked to predict the flow structure at values of the Rayleigh number substantially larger than the critical. At any rate, it would appear that the effect of the temperature dependent viscosity which, as shown by Pahn et *al.,* increases as l^{-6} , is sufficiently pronounced at these small depths to overcome the influence of the cylindrical geometry and thus produce hexagons. Also, as can be seen from Table 1, there is good agreement between the experimentally determined conditions at the point of instability and the theoretical predictions obtained by Liang [17] by a numerical solution of the appropriate linear stability equations, although the corresponding wave numbers differ by about 10 per cent.

Experiments were also performed in which the Aroclor was separated from the glass plate by a 1.0 mm thick air-gap. Off hand, one might expect that the onset of convection would be caused here primarily by surface-tension gradients, as was the case in experiment no. 2. However, since the surface of Aroclor is easily contaminated by impurities (the surface tension of pure Aroclor was 44.4 dyn cm- i, that of Aroclor contaminated with aluminum powder was 39.6 dyn cm^{-1}) it can be shown on theoretical grounds [18] that their presence here serves to dampen convection due to surface-tension effects. (In the case of silicone oil the addition of impurities had no measurable effect on its surface tension.) Hence, the experiments were interpreted on the assumption that the motion was due to buoyancy forces alone.

Two liquid depths were investigated. In the first case, with $l = 6.92$ mm, concentric rolls were produced such as those shown in Fig. 5; with a smaller liquid depth, $l = 3.99$ mm, stable hexagons were formed, seen in Fig. 6. These experimental results were, in all respects, very similar to those discussed previously in which the liquid touched the glass plate. Also, as seen in Table 1, these again compare reasonably well with the theoretically predictions when $l = 6.92$ mm. At the lower depth, however, there

appears to be a definite disagreement not only in the wave number which, as in experiment no. 4, was somewhat larger than expected, but especially in the measured critical Rayleigh number which was smaller than the theoretical value by a factor of 2.5 . This last discrepancy is probably due to the fact that the theoretical value for *R was* estimated on the assumption that, owing to the accumulation of surface active agents, the air-liquid interface was stagnant and that surface-tension gradients did not contribute to the convection Clearly, lower critical Rayleigh and wave numbers would have resulted had these assumptions been relaxed. This particular point, though, deserves further study. Also, it is not clear why the observed size of the hexagons in experiments 4 and 6 was, in both cases, consistently smaller than that predicted theoretically.

4. CONCLUSIONS

The existence of a hexagonal flow pattern in thin layers of a liquid having a temperature sensitive viscosity and undergoing buoyancydriven convection has been verified experimentally. In contrast to some theoretical predictions, these hexagonal cells, which were somewhat smaller than expected, were found to remain stable over a range of temperature differences extending to at least three times the critical, thus suggesting that the analysis by Palm et al. [9] may not remain valid much beyond the critical point where, in fact, it is meant to be applied. For relatively thick layers, the pattern was observed to consist of concentric rolls. Thus, depending on the liquid depth, either hexagons or rolls, but not both, were generally found to form throughout the range of temperature differences investigated. This, again, runs counter to recent theoretical results $[6-9]$, according to which, in such cases, the motion should always consist of hexagons rather than rolls in the immediate neighborhood of the critical point. It is suggested that this discrepancy may be due to secondary factors, such as the shape of the container, which, near

Table 1. Comparison *between experimental results and theoretical predictions*

(a) Ref. $[15]$; (b) Ref. $[3]$; (c) Ref. $[17]$; (d) estimated value.

the point of instability, may be sufficiently important to prevent the formation of the hexagons whenever the liquid layer is deep enough so as to render the effects of the variable viscosity less pronounced.

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REFERENCES

- 1. S. **CHANDRASEKHAR,** *Hydrodynamic and Hydromagnetic Stability.* Clarendon Press, Oxford (1961).
- 2. J. R. A. PEARSON, On convection cells induced by surfac tension, J. *Fluid Mech. 4, 489* (1958).
- 3. D. A. **NIELD,** Surface tension and buoyancy effects in cellular convection, J. Fluid *Mech.* 19, 341 (1964).
- 4. J. W. **SCALON** and .L. A. SEGEL, Finite amplitude convection induced by surface tension, *J. Fluid Mech.* 30, 149 (1967).
- 5. A. **SCHL~TER,** D. LORTZ and F. H. Buss& On the stability of steady finite amplitude convection, *J. Fluid Mech. 23,* 129 (1965).
- 6. E. PALM, On the tendency towards hexagonal cells in steady convection, *J. Fluid Mech.* 8, 183 (1960).
- 7. L. A. SEGE and J. T. STUART, On the questlon of the preferred mode in cellular thermal convection, *J. Fluid Me&. 13, 289 (1962).*
- 8. F. H. Busse, The stability of finite amplitude con vection and its relation to an extremum principle, *J. Fluid Mech.* 30, 625 (1967).
- 9. E. PALM, T. ELLINGSEN and B. GJERIK, On the occur ence of cellular motion in B²nard convection, *J. Fluid Mech. 30, 651 (1967).*
- 10. Ernst Schmidt and P. L. Silveston, Natural con vection in horizontal liquid layers, *Chem.* Engng *Prog. Symp. Ser.* 55, No. 29, 163 (1959).
- 11. E. L. KOSCHMIEDER, On convection on a uniformly heated plane, *Beitr. Phys. Atmos. 39.* 1 (1966).
- 12. E. L. KOSCHMIEDER, On convection under an air surface, *J. Fluid Mech. 30, 9 (1967).*
- 13. M. M. CHEN and S. A. WHITEHEAD, Evolution of two dimensional periodic Rayleigh convection cells of arbitrary wave-numbers, *J. Fluid Mech. 31.* 1 (1968).
- 14. S. H. DAVIS, Convection in a box: Linear theory, *J. Fluid Mech. 30, 465 (1967).*
- 15. E. M. Sparrow, R. J. Goldstein and V. K. Jonnson, Thermal instability in a horizontal fluid layer: Effect or boundary conditions and non-linear temperature profile, *J. Fluid Mech.* 18, 513 (1964).
- 16. **RUBY KRISHNAMURTI,** Finite amplitude convection with changing mean temperature, *J. Fluid Mech. 33, 445 (1968).*
- 17. *S.* F. **LANG,** Ph.D. Thesis, Stanford University (1969).
- 18. **J. C. BERG** and **A. ACRIVOS,** The effect of surface active agents on convection cells induced by surface tension, *Chem. Engng Sci. 20, 737* (1965).

Table 1. (continued)

EXPERIENCES SUR LA STRUCTURE CELLULAIRE DANS LA CONVECTION DE BENARD

Résumé-La convection naturelle dans une cellule cylindrique a été étudiée expérimentalement par endessous en employant deux fluides différents. Dans l'ensemble, un bon accord a été obtenu avec de nombreuses prévisions théoriques. Dans l'huile de silicone, dont les propriétés physiques sont relativement insensibles a la temperature, on a trouve que la configuration de l'ecoulement consistait en rouleaux concentriques dam le cas de la convection due a la pesanteur, et de celhdes hexagonales pour l'ecoulement dû à la tension superficielle. Ces observations sont compatibles avec celles publiées récemment par Koschmieder. Avec de l'Aroclor, dont la viscosité dépend fortement de la température, les rouleaux apparaissent pour des profondeurs de liquide de 6 mm et des hexagones pour des profondeurs de 5 mm. PlutGt que de se transformer en rouleaux comme certaines theories l'avaient predit, ces hexagones, qui étaient quelque peu plus petits qu'attendu, restaient stables dans une gamme de différences de températures s'etendant jusqu'au moins trois fois la valeur critique. Ceci suggere que les analyses non-lineaires disponibles actuellement ne devraient pas être extrapolées beaucoup plus loin que le point critique, où en fait elles doivent s'appliquer.

VERSUCHE ÜBER DIE ZELLULARSTRUKTUR BEI BENARD-KONVEKTION

Zusammenfassung-Die natürliche Konvektion in einer zylindrischen Zelle wurde experimentell untersucht an zwei verschiedenen Flüssigkeiten. Insgesamt wurde gute Übereinstimmung mit vielen theoretischen Vorhersagen gefunden. In Silikonöl, dessen Stoffwerte relativ unempfindlich gegen Temperaturänderung sind, zeigte sich, dass das Strömungsmuster im Fall der auftreibsbedingten Konvektion aus konzentrischen Rollen bestand und im Fall der oberflächenspannungsbedintten Strömung aus hexagonalen Zellen, Diese Beobachtungen stimmen mit den kürzlich von Koschmieder beschriebenen überein. Bei Aroclor, dessen Zähigkeit stark temperaturabhängig ist, erschienen Rollen für Flüssigkeitshöhen von 6 mm und Sechsecke für Höhen von 5 mm. Statt in Rollen überzugehen, wie einige Theorien verlangen, bleiben diese Sechsecke die etwas kleiner sind als erwartet, stabil über einen Bereich von Temperaturdifferenzen, der sich auf mindestens dreimal den kritischen erstreckt. Diese Beobachtung verbietet die Extrapolation von gegenwärtig verfügbaren nicht-linearen Analysen weiter über den kritischen Punkt hinaus, wofür sie eigentlich anwendbar sein sollten.

ЭКСПЕРИМЕНТЫ ПО ИССЛЕДОВАНИЮ ЯЧЕИСТОЙ СТРУКТУРЫ ПРИ КОНВЕКЦИИ БЕНАРДА

Анноация--Экспериментально исследовалась естественная конвекция в цилиндрической ячейке при использовании двух различных жидкостей. В целом, получено хорошее согласование с многими теоретическими расчетами. При использовании силоксанового масла, физические свойства которого мало зависят от изменений температуры, найдено, что картина течения состоит из концентрических валов в случае конвекции, вызванной силами плавучести, и из шестиугольных ячеек при течении, вызванном силами поверхностного натяжения. Эти наблюдения находятся в соответствии с наблюдениями, доложенными недавно Кошмидером. При использовании арохлора, вязкость которого сильно зависит от температуры, валы появлялись при толщине слоя жидкости 6 мм. а шестиугольники-при толщине слоя жидкости 5 мм. Найдено, что шестиугольники, размер которых был несколько меньше, чем предполагалось, не превращались в валы, нак это предсказывается по некоторым теориям, а оставались стабильными в диапазоне разности температур, превышающих, по крайней мере, в три раза критическое значение температуры. Это свидетельствует о том, что имеющиеся в настоящее время нелинейные анализы нельзя экстраполировать значительно выше критической точки, где предполагается их фактическое использование.